2025

(FYUGP)

(5th Semester)

MATHEMATICS

(MINOR)

Paper Code : MAT/M5

(Riemann Integration and Series of Functions)

Full Marks : 75 *Pass Marks* : 40%

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* If P_1, $P_2 \in P[a, b]$; then prove that

(i) $L(P_1, f) \leq U(P_2, f)$

(ii) $L(P_2, f) \leq U(P_1, f)$

where $f : [a, b] \to \mathbb{R}$ be a bounded function. 2½+2½=5

(b) Prove that a constant function f is Riemann integrable, where $f : [a, b] \to \mathbb{R}$. 5

OR

2. *(a)* Show that

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{1}{8n}\right]=\frac{3}{8}$$ 5

(b) If $f:[a,b]\to R$ is continuous on $[a,b]$, prove that f is integrable on $[a,b]$. 5

3. *(a)* Evaluate : 5

$$\int(3x-2)\sqrt{x^2-x+1}\,dx$$

(b) If f is continuous on $[a,b]$ and

$$\phi(t)=\int_a^t f(x)\,dx$$

then prove that

$$\phi'(t)=f(t)\ \forall\ t\in[a,b]$$ 5

OR

4. *(a)* If $f\in R[a,b]$ and $k\in R$, where $k>0$, then show that $kf\in R[a,b]$ and

$$\int_a^b (kf)(x)\,dx=k\int_a^b f(x)\,dx$$ 5

(b) Prove that

$$\frac{2\pi^2}{9}\le\int_{\pi/6}^{\pi/2}\frac{2x}{\sin x}\,dx\le\frac{4\pi^2}{9}$$ 5

5. *(a)* Show that the sequence $\{f_n\}$ of functions, where

$$f_n(x)=\frac{n}{x+n}$$

is uniformly convergent in $[0,k]$ whatever k may be but not uniformly convergent in $[0,\infty)$. 5

(b) For the sequence of functions $\{f_n\}$, where $f_n(x)=nx(1-x^2)^n$, $x\in[0,1]$, show that the limit of sequence of integrals is not equal to the integral of the limit of sequence of functions $(f_n)_{n=1}^{\infty}$. 5

OR

6. *(a)* Prove that a sequence $\{f_n\}$ of functions defined in an interval I is uniformly convergent on I iff for each $\varepsilon>0$ and $\forall x\in I$, there exists a positive integer

$$m\ni|f_{n_1}(x)-f_{n_2}(x)|<\varepsilon\ \forall n_1,n_2\ge m$$ 6

(b) Verify uniform convergence of the sequence $\{f_n\}$, where $f_n(x)=nxe^{-nx^2}$. 4

7. *(a)* Verify uniform convergence of the series

$$\sum_{n=1}^{\infty}\frac{x}{n(1+nx^2)}$$ 7

(b) Show that the series

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$

converges uniformly for all real values of x. 3

OR

8. *(a)* Prove that a number l is the limit superior of a bounded sequence $\langle f_n \rangle$ if and only if the following conditions are satisfied : 5

(i) For each $\varepsilon > 0$, $f_n > l - \varepsilon$ for infinitely many values of n.

(ii) For each $\varepsilon > 0$, $f_n < l + \varepsilon$ for all except finitely many values of n.

(b) If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and

$$\lim_{n \to \infty} |a_n| = \frac{1}{R}$$

then prove that $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$. 5

9. *(a)* If $f : [a, b] \to R$ is a bounded function and $P \in P(a, b)$, then show that

(i) $L(P, f) \leq U(P, f)$

(ii) $L(P, -f) = -U(P, -f) = -L(P, f)$ 5

(b) Evaluate : 5

$$\int \frac{x+14}{(x+13)^{10}} e^x \, dx$$

OR

10. *(a)* Examine for uniform convergence and continuity of the limit function of the sequence $\{f_n\}$, where

$$f_n(x) = \frac{1}{1+nx}, \quad x \in [0, 1]$$ 5

(b) Find the radius of convergence and the exact interval of convergence of the power series

$$\sum \frac{(n+1)}{(n+2)(n+3)} x^n$$ 5

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Subject Code : Ba/Bs/MAT/M5

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA 5th Semester End Term Examination, **2025 (FYUGP)**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Booklet No. A 1

Date Stamp ....

....

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA 5th Semester End Term Examination, **2025 (FYUGP)**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

DESCRIPTIVE TYPE

Booklet No. B ....

Signature of Invigilator(s)

/193

2025

(FYUGP)

(5th Semester)

MATHEMATICS

(MINOR)

Paper : MAT/M5

(Riemann Integration and Series of Functions)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×15=15

1. If $f:[a,b]\to R$ is a bounded function, then

(a) $\int_{\underline{a}}^{b} f(x)\,dx = \int_{a}^{\overline{b}} f(x)\,dx$ ( )

(b) $\int_{\underline{a}}^{b} f(x)\,dx \le \int_{a}^{\overline{b}} f(x)\,dx$ ( )

(c) $\int_{\underline{a}}^{b} f(x)\,dx \ge \int_{a}^{\overline{b}} f(x)\,dx$ ( )

(d) $\int_{\underline{a}}^{b} f(x)\,dx \ne \int_{a}^{\overline{b}} f(x)\,dx$ ( )

2. Let $f : [a, b] \to R$ be a bounded function. Which of the following conditions is sufficient to guarantee that f is Riemann integrable on $[a, b]$?

(a) f has only finitely many points of discontinuity on $[a, b]$ ( )

(b) f is monotonic on $[a, b]$ ( )

(c) f is continuous everywhere on $[a, b]$ ( )

(d) All of the above ( )

3. Let $P = \{x_0, x_1, \cdots, x_n\}$ be a partition of the interval $[a, b]$. A partition P' is said to be a refinement of P if

(a) P' has fewer points than P ( )

(b) P' contains all the points of P and possibly some additional points ( )

(c) P' divides $[a, b]$ into equal subintervals ( )

(d) the mesh of P' is smaller than the mesh of P ( )

4. The Riemann integral of a bounded function f on $[a, b]$ is defined as

(a) the limit of the average value of f at the end points ( )

(b) the limit of a sum of the form $\Sigma f(x_r)\delta_r$, if the limit exists ( )

(c) the difference between the maximum and minimum values of f on $[a, b]$ ( )

(d) the area of the rectangle with base $[a, b]$ and height $f(b)$ ( )

5. If f is bounded and Riemann integrable on $[a, b]$ and $c \in [a, b]$, then

(a) f is integrable only on $[a, c]$ ( )

(b) f is integrable only on $[c, b]$ ( )

(c) f is integrable on both $[a, c]$ and $[c, b]$ ( )

(d) Nothing can be said about integrability on subintervals ( )

6. If f is bounded and integrable on $[a, b]$, then

(a) the positive and negative parts of f are also integrable on $[a, b]$ ( )

(b) only the positive part of f is integrable ( )

(c) only the negative part of f is integrable ( )

(d) neither positive nor negative part of f is integrable ( )

7. Which of the following is not true about uniform convergence?

(a) If $f_n \to f$ uniformly and each f_n is continuous, then f is continuous ( )

(b) Uniform convergence preserves integration on closed bounded intervals ( )

(c) Uniform convergence preserves differentiation always ( )

(d) uniform convergence implies pointwise convergence ( )

8. Let $f_n(x) = x^n$ on $[0,1]$, then $\{f_n\}$

(a) converges pointwise to $f(x) = 0 \ \forall \ x \in [0,1]$ ( )

(b) converges pointwise to $f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & x = 1 \end{cases}$ ( )

(c) converges uniformly to $f(x) = 0$ ( )

(d) converges uniformly to $f(x) = 1$ ( )

9. Let $f_n(x) = \sin\left(\frac{x}{n}\right)$ on R, then $\{f_n\}$

(a) converges pointwise to $f(x) = 0$ but not uniformly ( )

(b) converges uniformly to $f(x) = 0$ ( )

(c) converges pointwise to $f(x) = \sin x$ ( )

(d) does not converge ( )

10. Which of the following statements is true?

(a) Pointwise limit of continuous functions is also continuous. ( )

(b) Uniform limit of continuous functions is continuous. ( )

(c) Both *(a)* and *(b)* ( )

(d) Neither *(a)* nor *(b)* ( )

11. If f is continuous on $[a, b]$ and

$$F(x) = \int_a^b f(t)\, dt$$

then

(a) $F'(x) = f(a)\ \forall\ x \in [a, b]$ ( )

(b) $F'(x) = f(x)\ \forall\ x \in [a, b]$ ( )

(c) $F'(x) = \int_a^x f(t)\, dt$ ( )

(d) $F'(x) = 0\ \forall\ x \in [a, b]$ ( )

12. For the given sequence

$$\left\langle (-1)^n \left(1 + \frac{1}{n}\right) \right\rangle$$

which of the following is correct?

(a) Limit superior = limit inferior ( )

(b) Neither limit superior nor limit inferior exists ( )

(c) Limit superior $= 1$ and limit inferior $= -1$ ( )

(d) Limit superior $= 1$ and limit inferior $= 0$ ( )

13. The series $\sum r^n \cos nx$ converges uniformly for

(a) $0 < r < 1$ ( )

(b) $0 \le r < 1$ ( )

(c) $0 \le r \le 1$ ( )

(d) None of the above ( )

14. Both the series

$$\sum \frac{\sin nx}{n^2} \text{ and } \sum \frac{\cos nx}{n^2}$$

are

(a) uniformly convergent in $[0, \pi]$ ( )

(b) uniformly convergent in $[0, 2\pi]$ ( )

(c) uniformly convergent in $[\pi, 2\pi]$ ( )

(d) None of the above ( )

15. The two power series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=1}^{\infty} n a_n x^{n-1}$ have

(a) different radius of convergence ( )

(b) same radius of convergence ( )

(c) reciprocal radius of convergence ( )

(d) None of the above ( )

SECTION—II

(*Marks* : 10)

Answer any *five* of the following : 2×5=10

1. Compute $L(P, f)$ for $f(x) = x^2$ on $[0,1]$ and $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$.

2. Find the primitive of f, where $f(x) = x\cos x$ on $\left[0, \dfrac{\pi}{2}\right]$.

3. If f is a non-negative continuous function on $[a, b]$ and if $f(c) > 0$ for some $c \in [a, b]$, then prove that

$$\int_a^b f(x)\,dx > 0$$

4. Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{nx}{1+n^2x^2}, \quad x \in [0,1]$$

can be differentiated term-by-term at $x = 0$.

5. Define pointwise and uniform convergence. State the difference between them.

6. The series $\sum \frac{(-1)^{n+1}}{n} |x|^n$ is uniformly convergent in $[-1,1]$. Prove or disprove.

7. If a power series diverges for $x = x'$, then it converges for every $x = x''$, where $|x''| > |x'|$. Prove or disprove.

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