2025

(FYUGP)

(3rd Semester)

PHYSICS

(MINOR)

Paper Code : PHYM-301(T)

(Mathematical Physics—II)

Full Marks : 75 *Pass Marks* : 40%

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Group

GROUP—A

1. *(a)* Determine the Fourier series of the function

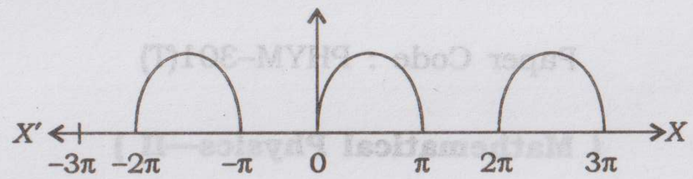
$$f(x)=\begin{cases}-1 & \text{for} \quad -\pi < x < -\pi/2 \\ 0 & \text{for} \quad -\pi/2 < x < \pi/2 \\ +1 & \text{for} \quad \pi/2 < x < \pi\end{cases}$$ 6

(b) Derive the expansion for the function

$$f(x) = \cos(rx), \quad -\pi \le x \le \pi$$

where r is a function. 4

2. Derive and expand the Fourier series for the following waveform : 10



GROUP—B

3. Derive the solution for the Hermite's equation when $m = 0$. 10

4. *(a)* Solve the differential equation

$$2x^2y'' + xy' - (x+1)y = 0$$

in power series about $x = 0$. 4

(b) Evaluate the different equation

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

using Frobenius method. 6

GROUP—C

5. *(a)* Prove that

$$P_n(x) = \frac{1}{2^n \cdot n!}\frac{d^n}{dx^n}(x^2-1)^n$$ 6

(b) Express $1+2x-3x^2+4x^3$ in terms of Legendre polynomials. 4

6. *(a)* Prove the orthogonality of Legendre polynomials. 5

(b) Determine the value of

$$\int_{-1}^{+1}(x^2-1)P_{n+1}P_n'\,dx$$ 5

GROUP—D

7. *(a)* Using the method of separation of variables, determine the solution of

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

and $u = e^{-5y}$ when $x = 0$. 5

(b) Derive the one-dimensional partial differential wave equation. 5

8. Prove that

$$\nabla^2 V = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial^2 V}{\partial r^2} + \frac{\partial^2 V}{r^2 \partial \theta^2}$$ 10

GROUP—E

9. *(a)* Evaluate the beta function

$$\beta(l, m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$$ 5

(b) Explain the reporting error using standard deviation. 5

10. *(a)* Prove that

$$\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}}\,dx = \frac{1}{5005}$$ 4

(b) Determine the integral

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{dx}{(1+x^4)}$$ 6

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Subject Code : Bs/PHYM-301(T)

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA
3rd Semester End Term Examination, **2025 (FYUGP)**

Subject .....

Paper .....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Booklet No. A 44

Date Stamp .....

.....

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA

3rd Semester End Term

Examination, **2025 (FYUGP)**

Roll No. .....

Regn. No. .....

Subject .....

Paper .....

DESCRIPTIVE TYPE

Booklet No. B .....

Signature of Invigilator(s)

/117

2025

(FYUGP)

(3rd Semester)

PHYSICS

(MINOR)

Paper Code : PHYM–301(T)

(Mathematical Physics—II)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×15=15

1. If a periodic function $f(x)$ can be expanded in a Fourier series, then the series consists of which of the following?

I. A constant term a_0

II. A component at the fundamental frequency that can be determined by a_1, b_1

III. Components of harmonics determined by a_2, a_3, ..., b_2, b_3

(a) Only I ( )

(b) Only II ( )

(c) Both II and III ( )

(d) All I, II, III ( )

2. If $f(x) = x^4$ in the interval (–1, 1), then the Fourier coefficient b_n will be equal to

(a) zero ( )

(b) 2π ( )

(c) $2\pi / 3$ ( )

(d) $2\pi / n$ ( )

3. The Fourier series for an odd function contains only

(a) terms in cosine ( )

(b) terms in sine ( )

(c) terms in sine and cosine ( )

(d) None of the above ( )

4. If $x = 0$ is a regular singularity of the differential equation $y'' + P(x)y' + Q(x)y = 0$, then the series solution is given by

(a) $\sum_{k=0}^{\infty} ax^k$ ( )

(b) $\sum_{k=0}^{\infty} a_k x^k$ ( )

(c) $\sum_{k=0}^{\infty} a_k x^{m+k}$ ( )

(d) $\sum_{k=0}^{\infty} a_k x^{m-k}$ ( )

5. If $x = 1$ is an ordinary point of

$$(x-1)^2 \frac{d^2y}{dx^2} + 4(x-1)\frac{dy}{dx} + 4y = 0$$

which of the following is true?

I. $x = 1$ is also a regular point of the differential equation of $(x-1)^2 \frac{d^2y}{dx^2} + 4(x-1)\frac{dy}{dx} + 4y = 0$.

II. $x = 1$ is an irregular point of the differential equation of $(x-1)^2 \frac{d^2y}{dx^2} + 4(x-1)\frac{dy}{dx} + 4y = 0$.

(a) Only I ( )

(b) Only II ( )

(c) Both I and II ( )

(d) Neither I nor II ( )

6. The solution of Laguerre's equation

$$xy'' + (1-x)y' + ny = 0$$

is given by

(a) $L_n(x) =$

$$(1)^n\left[x^n + \frac{n^2}{1!}x^{n-1} + n^2\frac{(n+1)^2}{2!}x^{n+2} + \cdots + (1)^n n!\right]$$

( )

(b) $L_n(x) =$

$$(-1)^n\left[x^n - \frac{n^2}{1!}x^{n-1} + n^2\frac{(n-1)^2}{2!}x^{n-2} + \cdots + (-1)^n n!\right]$$

( )

(c) $L_n(x) =$

$$(1)^{-n}\left[x^{-n} + \frac{n^2}{1!}x^{-(n+1)} + n^2(n-1)^{-(n+2)} + \cdots + (-1)^n n!\right]$$

( )

(d) $L_n(x) =$

$$(-1)^{-n}\left[x^{-n} + \frac{n^2}{1!}x^{-(n-1)} + n^2\frac{(n-1)^2}{2!}x^{-(n-2)} + \cdots + (1)^{-n} n!\right]$$

( )

7. The coefficient of 2^n in the expansion of $(1-2xz+z^2)^{-1/2}$ in ascending power of x is

(a) $P_n(x)$ ( )

(b) $P_n(z)$ ( )

(c) $P_n(x)^{-1/2}$ ( )

(d) $P_n(z)^{-1/2}$ ( )

8. The value of $P'_{n+1} + P'_n$ is equal to

(a) $2P_0 + 4P_1 + 6P_2 + \ldots + 2nP_n$ ( )

(b) $P_0 + 2P_1 + 3P_2 + \ldots + (n+1)P_n$ ( )

(c) $P_0 + 3P_1 + 5P_2 + \ldots + (2n+1)P_n$ ( )

(d) $P_0 + 4P_1 + 7P_2 + \ldots + (3n+1)P_n$ ( )

9. $J_{\frac{1}{2}}(x)$ is given by

(a) $\sqrt{\frac{2}{\pi x}} \sin x$ ( )

(b) $\sqrt{\frac{\pi}{2x}} \cos x$ ( )

(c) $\sqrt{\frac{2\pi}{x}} \sin x$ ( )

(d) $\sqrt{\frac{2\pi}{x}} \cos x$ ( )

10. Laplace's equation is polar coordinate is given by

(a) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = 0$ ( )

(b) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$ ( )

(c) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} = 0$ ( )

(d) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$ ( )

11. The general solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial t^2} = 0$$

is

(a) $(C_1 \cos px + C_2 \sin px)(C_3 \cos pt + C_4 \sin pt)$ ( )

(b) $(C_1 \sin px + C_2 \sin px)(C_3 \cos pt + C_4 \cos pt)$ ( )

(c) $(C_1 \cos px + C_2 \cos px)(C_3 \sin pt + C_4 \sin pt)$ ( )

(d) $(C_1 \sin px + C_2 \cos px)(C_3 \cos pt + C_4 \sin pt)$ ( )

12. Transverse vibration in one dimensional wave equation, the motion in horizontal direction is

(a) zero ( )

(b) unity ( )

(c) infinity ( )

(d) None of the above ( )

13. The value of the $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}}\, dx$ is equal to

(a) $\frac{2}{3}\sqrt{\pi}$ ( )

(b) $\frac{3}{2}\sqrt{\pi}$ ( )

(c) $\frac{1}{3}\sqrt{\pi}$ ( )

(d) $\frac{1}{2}\sqrt{\pi}$ ( )

14. An error is random, if

(a) repeated measurements result in error with different magnitudes and signs ( )

(b) repeated measurements result in error with same magnitude and sign ( )

(c) repeated measurements taken under the same condition yield same error in magnitude and sign ( )

(d) repeated measurements taken under different conditions yield different errors in magnitude and sign ( )

15. If a car travels at a speed of 100 ± 7 km/h for time $t = 2 \cdot 5 \pm 0 \cdot 1$ hours, then the relative error in the distance travelled will be

(a) 0·07 ( )

(b) 0·11 ( )

(c) 0·15 ( )

(d) 0·17 ( )

SECTION—II

(*Marks* : 10)

Answer any *five* of the following : 2×5=10

1. Determine the half-range Fourier series for $f(x) = e^x$ in the interval $0 < x < 1$.

2. What is an even function? Illustrate with an example.

3. Write the Bassel's equation. State two applications of Bessel's function.

4. $x = 0$ is a singular point of $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 4y = 0$.

Justify.

5. Show that $\int_{-1}^{+1} P_n(x)dx = 0,\ n \neq 0$ and $\int_{-1}^{+1} P_n(x)dx = 2,$ $n = 0.$

6. Prove that $\lim_{x \to 0} \frac{J_n(x)}{x^n} = \frac{1}{2^n \Gamma(n+1)}$, $(n > -1)$.

7. Show that $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is a two-dimensional wave equation.

8. Solve $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the condition $u'(0, t) = 0\ t > 0$.

9. Prove that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$$

10. Evaluate $erf(-x) = -erf(x)$.

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