

2024

(FYUGP)

(1st Semester)

MATHEMATICS

(Minor)

Paper Code : MAT/M1

(**Calculus**)

Full Marks : 75

Pass Marks : 40%

Time : 3 hours

(**PART : B—DESCRIPTIVE**)

(*Marks : 50*)

*The figures in the margin indicate full marks
for the questions*

1. (a) Find the domain and range of the following functions : 3+3=6

(i) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x^2}{1+x^2}$$

(ii) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = -|x|$

- (b) Let $f: A \rightarrow B$, $g: B \rightarrow A$ be two functions such that $g \circ f = I_A$. Prove that f is $|$ - and g is onto function. 4

OR

2. (a) Examine whether the given functions are bijective or not : 3+3=6

(i) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(x) = x^2 + 3$

(ii) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x-2}{x-3}$$

- (b) Let $f: A \rightarrow A$ be a function. If $f \circ f = I_A$, then prove that f is invertible and $f = f^{-1}$. 4

3. (a) Evaluate : 2½

$$\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1}$$

- (b) Let $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$. Prove f is continuous at 0. Using the ϵ - δ property of theorem. 2½

- (c) If a function $f(x)$ is continuous in the closed interval $[a, b]$ and $f(a) \neq f(b)$, then show that the function $f(x)$ assumes all the values between $f(a)$ and $f(b)$. 5

OR

4. (a) Describe the types of discontinuities that a function may have. What type of discontinuity the function

$$f(x) \begin{cases} \frac{x^4 + 4x^3 + 2x}{\sin x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

have?

3

- (b) A function f is defined as follows
 $f(x) = x^2 + 3x + 4$ on $[1, 2]$. Then show that f is continuous as well as uniformly continuous on $[1, 2]$.

3

- (c) Prove that—

(i) the limit of a function at a point, if it exists, is unique;

(ii) if

$$\lim_{x \rightarrow a} f(x) = l \text{ and } \lim_{x \rightarrow a} f(x) = l',$$

then $l = l'$.

4

5. (a) If f and g are both differentiable at x and if $g(x) \neq 0$, then show that $\frac{f}{g}$ is differentiable at x and

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

4

(b) Evaluate :

3+3=6

$$(i) \lim_{x \rightarrow 0} \frac{x e^x - \log(1+x)}{x^2}$$

$$(ii) \lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$$

OR

6. (a) Obtain the multiple roots of $x^5 - 10x^2 + 15x - 6 = 0$ and hence solve it completely. 4

(b) Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$. 6

7. (a) Evaluate any two of the following : 3×2=6

$$(i) \int \tan^4 x \, dx$$

$$(ii) \int \frac{dx}{(x+1)\sqrt{x^2+1}}$$

$$(iii) \int \frac{x}{1+\sin x} \, dx$$

(b) Test the convergence of the following :

2+2=4

$$(i) \int_0^{\infty} \frac{dx}{x^2 + 2x + 2}$$

$$(ii) \int_0^{\infty} \frac{dx}{(x+1)(x+2)}$$

OR

8. (a) Evaluate any *two* of the following : $3 \times 2 = 6$

(i) $\int \frac{\sin x}{\sin(x-a)} dx$

(ii) $\int e^{ax} \cos bx dx$

(iii) $\int \frac{1}{x^2 - a^2} dx$

- (b) Integrate the following : $2+2=4$

(i) $\int \frac{dx}{x(a + b \log x)}$

(ii) $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$

9. (a) Define a relation from a set A to set B .
Hence if R be the relation of $\mathbb{Z}$ defined by

$$R = \{ (a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}, a^2 = b^2 \}$$

Find—

(i) R ;

(ii) domain of R ;

(iii) range of R . 5

- (b) If a function f is uniformly continuous in a certain interval I , then show that it is necessarily continuous on I . 5

OR

10. (a) If $f(x)$ and $\phi(x)$ be two given functions of x capable of being expanded by Taylor's theorem and if

$$f(a) = \phi(a) = 0,$$

then show that

$$\lim_{x \rightarrow a} \frac{f(x)}{\phi(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{\phi'(x)} \quad 6$$

- (b) Integrate : 4

$$\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

Subject Code : Ba/Bs/MAT/M1

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA
1st Semester End Term

Examination, **2024 (FYUGP)**

Subject

Paper

INSTRUCTIONS TO CANDIDATES

1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.
2. This paper should be **ANSWERED FIRST** and submitted within **1 (one) Hour** of the commencement of the Examination.
3. While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.

Signature of
Scrutiniser(s)

Signature of
Examiner(s)

Booklet No. A 21

Date Stamp

To be filled in by the Candidate

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1st Semester End Term

Examination, **2024 (FYUGP)**

Roll No.

Regn. No.

Subject

Paper

DESCRIPTIVE TYPE

Booklet No. B

Signature of
Invigilator(s)

2024

(FYUGP)

(1st Semester)

MATHEMATICS

(Minor)

Paper Code : MAT/M1

(**Calculus**)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(Marks : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : $1 \times 10 = 10$

1. The function $y = \cos x$ is defined for

(a) $-1 \leq x \leq 1$ ()

(b) $(-\infty, -1]$ ()

(c) $[1, \infty)$ ()

(d) all x ()

2. If $A = \{1, 2\}$, $B = \{3, 4\}$, then the number of relations from A into B is

(a) 16 ()

(b) 8 ()

(c) 4 ()

(d) 0 ()

3. The function $f(x) = x^n$ where n is a negative odd integer is

(a) defined for all x ()

(b) defined for $x = 0$ ()

(c) undefined for all x ()

(d) undefined for $x = 0$ ()

4. The union $[a - \delta, a] \cup [a, a + \delta]$ which is the same as $[a - \delta, a + \delta] - \{a\}$ is called

(a) deleted neighbourhood of a ()

(b) deleted neighbourhood of δ ()

(c) deleted neighbourhood of $a - \delta$ ()

(d) deleted neighbourhood of $a + \delta$ ()

5. $\lim_{x \rightarrow 0} \frac{1}{|x|}$ is

(a) 0 ()

(b) ∞ ()

(c) $-\infty$ ()

(d) None of the above ()

6. If $\text{RHL} = \text{LHL} \neq \lim f(x)$, then $f(x)$ is said to have a

(a) discontinuity of first kind ()

(b) discontinuity of second kind ()

(c) mixed discontinuity ()

(d) removable discontinuity ()

7. If $y = \sin x^3$, then $\frac{dy}{dx} =$

(a) $3x^2 \cos x^3$ ()

(b) $3x^2 \sin x^3$ ()

(c) $3x^2 \cos x^2$ ()

(d) $3x^2 \sin x^2$ ()

8. The side of a square is increasing at the rate of 0.2 cm/s. Then the rate of increase of the perimeter of the square is

(a) 0.7 cm/s ()

(b) 0.8 cm/s ()

(c) 0.6 cm/s ()

(d) 0.4 cm/s ()

9. The value of $\int \frac{dx}{\sin^2 x \cos^2 x}$ is

(a) $\cot x - \tan x + c$ ()

(b) $\cot x + \tan x + c$ ()

(c) $\tan x - \sec x + c$ ()

(d) $\tan x + \sec x + c$ ()

10. The integral $\int \frac{\log x}{x} dx$ has the value

(a) $\frac{1}{2}(\log x)^2 + c$ ()

(b) $\frac{1}{2}(\log x) + c$ ()

(c) $(\log x)^2 + c$ ()

(d) $(\log x) + c$ ()

SECTION—II

(Marks : 5)

Fill in the blanks :

1×5=5

1. Logarithmic function is not defined for
2. The function $f(x) = \sqrt{x-1} + \sqrt{5-x}$ is defined at
3. The derivative of $\log(\log x)$ is
4. The value of $\int e^x \{f(x) + f'(x)\} dx$ is
5. If $f(x_0 + 0)$ and $f(x_0 - 0)$ both exist and are equal but this equal value is not equal to $f(x_0)$ is called

SECTION—III

(Marks : 10)

Answer the following questions (any five) :

2×5=10

1. Indicate true or false with a Tick (✓) mark and give justification.

If $f:R \rightarrow R$ be defined by $f(x) = x^2 - 3x + 2$, then $f \circ f:R \rightarrow R = I_R$.

True () False ()

Justification :

2. Indicate true or false with a Tick (✓) mark and give justification.

If $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x + 2$, then its inverse $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f^{-1}(x) = \frac{x-2}{3}$.

True () False ()

Justification :

3. Discuss the continuity of $f(x) = |x|$ at $x = 0$.

4. Find $\frac{dy}{dx}$ if $y = \sec(\tan^{-1} x)$.

5. Find the GCD of

$$f(x) = x^3 + x^2 - 16x + 20$$

$$\text{and } f'(x) = 3x^2 + 2x - 16$$

6. Evaluate $\int \frac{x}{x^4 - 9} dx$.
