

Ba/Bs/Mat/M1

2025

(FYUGP)

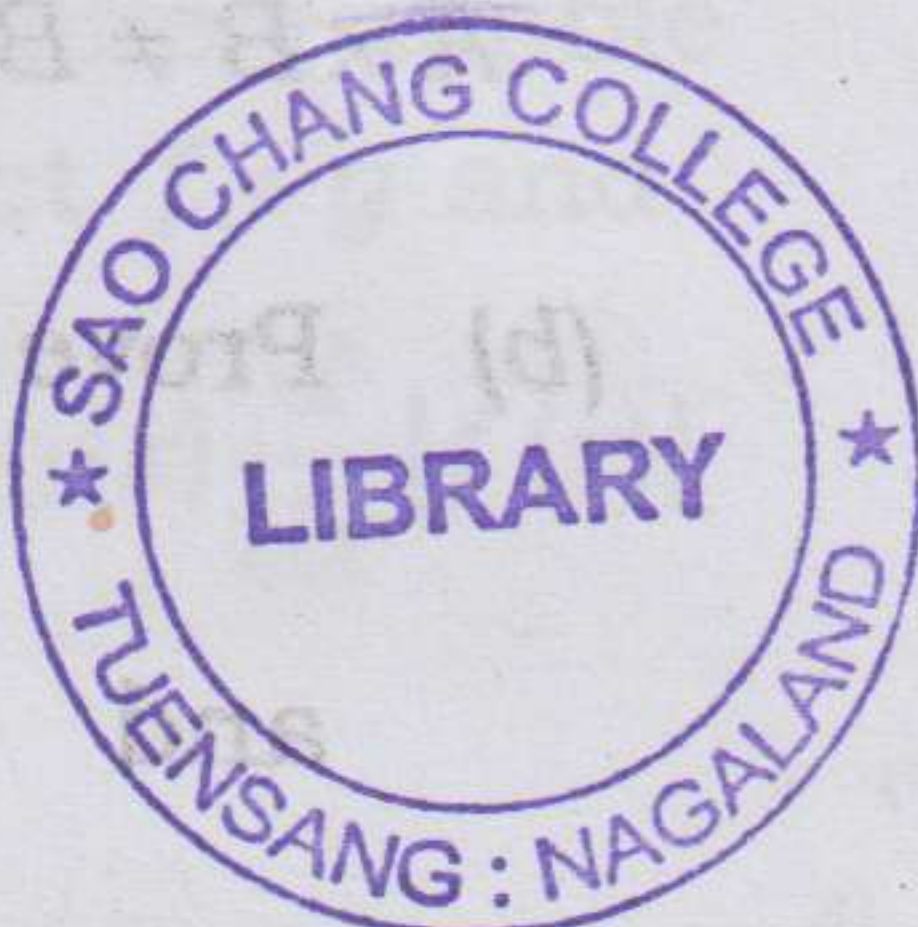
(1st Semester)

MATHEMATICS

(MINOR)

Paper Code : Mat/M1

(Calculus)



Full Marks : 75

Pass Marks : 40%

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

*The figures in the margin indicate full marks
for the questions*

1. (a) Define compound statements. Explain conjunction, disjunction, negation and biconditional statements with truth tables. 5

- (b) Using the standard laws, show that

$$\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r \quad 5$$

26L/43a

(Turn Over)

OR

2. (a) Using any example, prove that

$$A \times B \neq B \times A \text{ and } (A \times B) \times C \neq A \times (B \times C) \quad 5$$

- (b) Prove that

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$\text{and } (A \cup B) \cap C = A \cap (B \cap C) \quad 5$$

3. (a) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, where $f(x) = x^3 \forall x \in \mathbb{R}$ is one-one and onto. 5

- (b) If $f : A \rightarrow B$ and let I_A and I_B be identity functions of A and B respectively, then prove that

$$(f \circ I_A) = f \text{ and } (I_B \circ f) = f \quad 5$$

OR

4. (a) If $f : \mathbb{N} \rightarrow Y : f(x) = 4x^2 + 12x + 15$ and $Y = \text{range of } (f)$, then show that f is invertible and find f^{-1} . 5

- (b) If $A = \left\{ x : x \in \mathbb{R}, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \right\}$ and $B = \{ y : y \in \mathbb{R}, -1 \leq y \leq 1 \}$, then show that the function $f : A \rightarrow B$, where $f(x) = \sin x$ is invertible and hence find f^{-1} . 5

5. (a) Find an equation for the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$. 5

- (b) Prove that if f and g are differentiable at x , then so is the product $f \circ g$ and

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)] \quad 5$$

OR

6. (a) If $y = \sqrt{e^{\sin \log(x^2+7)^5}}$, using chain rule find $\frac{dy}{dx}$. 5

- (b) If $y \cdot \sqrt{x^2+1} = \log \{ \sqrt{x^2+1} - x \}$, then show that $(x^2+1) \frac{dy}{dx} + xy + 1 = 0$. 5

7. (a) Prove by summation that

$$\int_0^1 x^2 dx = \frac{1}{3} \quad 5$$

- (b) The circle $x^2 + y^2 = a^2$ revolves around the x -axis. Find the surface area and the volume of the whole surface generated. 5

OR

8. (a) Find the area of a hyperbola $xy = c^2$ bounded by the x -axis and the ordinates $x = a, x = b$. 5

- (b) Evaluate $\int \frac{2x+1}{\sqrt{3x+2}} dx$ by substitution method. 5

9. (a) Construct the truth table for the given statement

$$(p \Leftrightarrow q) \Leftrightarrow (r \rightarrow s)$$

5

- (b) If $f : [-1, 1] \rightarrow Y : f(x) = \frac{x}{(x+2)}$ and

$Y = \text{range } (f)$, then show that f is invertible and find f^{-1} . 5

OR

10. (a) If $y = e^x(\sin x + \cos x)$, then prove that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

5

- (b) Find the length of the arc of the curve

$$y = \frac{1}{2}a(e^{x/a} + e^{-x/a})$$

between the points $x = 0$ and $x = x$. 5

Subject Code : Ba/Bs/Mat/M1

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA
1st Semester End Term
Examination, 2025 (FYUGP)

Subject

Paper

INSTRUCTIONS TO CANDIDATES

1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.
2. This paper should be **ANSWERED FIRST** and submitted within **1 (one) Hour** of the commencement of the Examination.
- 3 While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.

Signature of
Scrutiniser(s)

Signature of
Examiner(s)



Booklet No. **A** 105

Date Stamp

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To be filled in by the
Candidate

BA / BSc / BCom / BBA / BCA
1st Semester End Term
Examination, 2025 (FYUGP)

Roll No.

Regn. No.

Subject

Paper

DESCRIPTIVE TYPE

Booklet No. B

Signature of
Invigilator(s)

2025

(FYUGP)

(1st Semester)

MATHEMATICS

(MINOR)

Paper Code : Mat/M1

(Calculus)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(Marks : 15)

Put a Tick (✓) mark against the correct answer in the brackets provided :

1×15=15

1. Which of the following is a statement?

- (a) What is your name? ()
- (b) $2 + 3 = 5$ ()
- (c) $x + 1 = 3$ ()
- (d) Open the door. ()

2. Which of the following is a tautology?

- (a) $P \vee \neg P$ ()
- (b) $P \wedge \neg P$ ()
- (c) $P \rightarrow \neg P$ ()
- (d) None of the above ()

3. The complement of universal set U is

- (a) U ()
- (b) ϕ ()
- (c) $\{0\}$ ()
- (d) None of the above ()

4. If $f = \{\{1\}, \{1, 2\}, \{2, 3\}\}$, then the union of the family f is

- (a) $\{1\}$ ()
- (b) $\{1, 2\}$ ()
- (c) $\{2, 3\}$ ()
- (d) $\{1, 2, 3\}$ ()

5. A function $f: A \rightarrow B$ is said to be into if

- (a) every element in B has at least a pre-image in A ()
- (b) every element in B has two pre-images in A ()
- (c) even a single element in B has no pre-image in A ()
- (d) None of the above ()

6. The function $f: R \rightarrow R: f(x) = x^2$ is

- (a) one-to-one and onto ()
- (b) one-to-one ()
- (c) neither one-to-one nor onto ()
- (d) None of the above ()

7. An invertible function has

- (a) a unique inverse ()
- (b) a common inverse ()
- (c) two inverses ()
- (d) None of the above ()

8. If $f : X \rightarrow Y$ and $A \subseteq X$, then $f(A) = \{f(x) : x \in A\}$ is called

(a) image of f in A ()

(b) image of A in f ()

(c) subset of A in f ()

(d) None of the above ()

9. The tangent line to the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is

(a) $y + f(x_0) = m_{\tan}(x - x_0)$ ()

(b) $y + f(x_0) = m_{\tan}(x + x_0)$ ()

(c) $y - f(x_0) = m_{\tan}(x - x_0)$ ()

(d) None of the above ()

10. A function $f(x)$ defined on (a, b) is said to be increasing if

(a) $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ ()

(b) $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ ()

(c) $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ ()

(d) $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ ()

11. The second derivative of $y = \log x$ is

(a) $\frac{1}{x}$ ()

(b) $\frac{1}{x^2}$ ()

(c) $-\frac{1}{x}$ ()

(d) $-\frac{1}{x^2}$ ()

12. In $\int f(x) dx = F(x) + C$, the integrand is

(a) $F(x)$ ()

(b) $f(x)$ ()

(c) dx ()

(d) $f(x) dx$ ()

13. $\int \frac{(1+x)^2}{x} dx =$

(a) $\log x + \frac{x^2}{2} + C$ ()

(b) $\log x + \frac{x}{2} + x^2 + C$ ()

(c) $\log x + \frac{x^2}{2} + 2x + C$ ()

(d) $\log x + 2x^2 + \frac{x}{2} + C$ ()

14. $r = 2a \sin \theta$ represents the equation of a/an

(a) circle ()

(b) cardioid ()

(c) parabola ()

(d) astroid ()

15. According to the fundamental theorem of integral calculus, where ϕ is the antiderivative of f

(a) $\int_a^b f(x) dx = f(b) - f(a)$ ()

(b) $\int_a^b f(x) dx = \phi(a) - \phi(b)$ ()

(c) $\int_a^b f(x) dx = f(a) - f(b)$ ()

(d) $\int_a^b f(x) dx = \phi(b) - \phi(a)$ ()

SECTION—II

(Marks : 10)

Answer any *five* of the following questions : $2 \times 5 = 10$

1. Prove or disprove $(p \wedge (p \Rightarrow q)) \rightarrow q$ is a tautology.

2. If $A = \{a, b\}$, $B = \{3, 6, 9\}$, then write all the elements of $P(A)$ and $P(B)$.

3. Show that $f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = 2x \quad \forall x \in \mathbb{N}$ is one-one and onto.

Answer any five of the following questions

2x5=10

1. Prove or disprove $(p \wedge (p \rightarrow q)) \rightarrow q$ is a tautology.

4. Show that the function $f(x) = (x^3 - 6x^2 + 12x - 18)$ is an increasing function on $\mathbb{R}$.

5. Evaluate $\int (a + bx)^n dx$.

6. Prove that

$$\int \frac{5(x-3)^2}{x\sqrt{x}} dx = \frac{10}{3} x^{3/2} - 60\sqrt{x} - 90x^{-1/2} + C$$
