2026

(FYUGP)

(4th Semester)

MATHEMATICS

(MINOR)

Paper : MAT/M4

(Numerical Methods)

Full Marks : 75

Pass Marks : 40%

Time : 3 hours

The figures in the margin indicate full marks for the questions.

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—I

(*Marks* : 15)

Choose the correct option : 1×15=15

1. The forward difference operator (Δ) is defined as
   (a) $\Delta f(x) = f(x-h) - f(x)$
   (b) $\Delta f(x) = f(x+h) - f(x)$
   (c) $\Delta f(x) = f(x) - f(x+h)$
   (d) $\Delta f(x) = f(x+h) + f(x)$

2. If $\Delta f(x) = 0$, then $f(x)$ is

(a) linear

(b) quadratic

(c) constant

(d) cubic

3. Which operator is most suitable for interpolation near the middle of the table?

(a) Δ

(b) ∇

(c) E

(d) δ

4. If $f(x) = 3x + 2$, $h = 1$, then $\Delta^2 f(x) =$

(a) 0

(b) 3

(c) 6

(d) 9

5. Gauss-Jordan method converts the matrix into _____ matrix.

(a) upper triangular

(b) lower triangular

(c) identity

(d) zero

6. Gauss-Jacobi method is _____ method.

(a) direct

(b) iterative

(c) graphical

(d) matrix inversion

7. Gauss-Seidel method converges faster than Gauss-Jacobi because

(a) it avoids division

(b) it uses updated values immediately

(c) it reduces matrix size

(d) it is direct method

8. Which method is most suitable for large systems of equation?

(a) Gauss-Jordan

(b) Gauss elimination

(c) Gauss-Jacobi

(d) Gauss-Seidel

9. Simpson's $\frac{3}{8}$ rule requires number of sub-intervals to be multiple of

(a) 1

(b) 2

(c) 3

(d) 4

10. The degree of Lagrange's polynomial for n data points is

(a) $n-1$

(b) n

(c) $n+1$

(d) n^2

11. Newton-Gregory formula requires data to be

(a) unequally spaced

(b) random

(c) equally spaced

(d) non-numeric

12. The error in Newton-Raphson method decreases if

(a) $\left|(x_1 - r)^2 \frac{M}{m}\right| \le 1$

(b) $\left|(x_1 - r^2) \frac{M}{m}\right| < 1$

(c) $\left|(x_1 - r^2) M\right| \le 1$

(d) $\left|(x_1 - r^2) M\right| < 1$

13. In Aitken's Δ^2 method, we can get the new approximate value of the roots in case

(a) one approximation is known

(b) two successive approximations are known

(c) three successive approximations are known

(d) four successive approximations are known

14. The method of tangents is also known as _____ method.

(a) bisection

(b) Newton-Raphson

(c) regula falsi

(d) Bolzano

15. In bisection method of solving an equation, the equation

(a) should be bounded

(b) should be continuous

(c) should be differentiable

(d) should be integrable

SECTION—II

(*Marks* : 10)

Answer any *five* of the following questions : 2×5=10

1. Prove or disprove the real root of the equation $f(x) = x^3 - x - 1 = 0$ lies in $[1, 1 \cdot 5]$ after the first approximation.

2. Justify that Aitken's Δ^2 method accelerates the rate of convergence.

3. Show that $y_3 = y_2 + \Delta y_1 + \Delta^2 y_0 + \Delta^3 y_0$.

4. Evaluate $\Delta^2 f(x)$ if $f(x) = \dfrac{1}{x(x+4)(x+8)}$.

5. State the principal part of errors in trapezoidal rule and Simpson's $\frac{3}{8}$ rule.

6. Evaluate $\int_{1/2}^{1} \frac{dx}{x}$ by trapezoidal rule dividing the range into 4 equal parts.

7. State the sufficient condition for Gauss-Seidel method to converge.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

1. With the usual notations, prove the following :

   (a) $\Delta^n e^x = (e-1)^n e^x$, where $h = 1$

   (b) $\frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}} = \Delta$

   (c) $\frac{\Delta^2}{E}\sin(x+h) + \frac{\Delta^2 \sin(x+h)}{E \sin(x+h)}$
   $= 2(\cos h - 1)\,[\sin(x+h) + 1]$

   3+3+4=10

Or

2. *(a)* Form a table of backward differences of the function

   $$f(x) = x^3 - 3x^2 - 5x - 7 \text{ for } x = -1, 0, 1, 2, 3, 4, 5$$

   5

   (b) If $y_0 = 3$, $y_1 = 12$, $y_2 = 81$, $y_3 = 2000$, $y_4 = 100$, then show that

   $$\Delta^4 y_0 = -7459$$

   5

3. *(a)* Find a negative root of the equation $\sin x = 1 + x^3$ by Newton's method. 5

(b) Compute the real root of $x^3 - 5x + 3 = 0$ in the interval [1, 2] by the False method. 5

Or

4. *(a)* Find the square root of 20 correct to 2 decimal places by Newton's method. 5

(b) Solve the equation $xe^x = 3$ by regula-falsi method starting with $a = 1$ and $b = 1 \cdot 5$ correct to 3 decimal places. 5

5. Find the inverse of the matrix

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$$

using—

(a) Gauss elimination method;

(b) Gauss-Jordan method. 5+5=10

Or

6. Solve the following system of equations by Gauss-Jacobi and Gauss-Seidel methods correct to 2 decimal places :

$$x + 3y + 52z = 173 \cdot 61$$
$$x - 27y + 2z = 71 \cdot 31$$
$$41x - 2y + 3z = 65 \cdot 46$$

with initial approximations $x = 1$, $y = -2$, and $z = 3$. 5+5=10

7. *(a)* Use Lagrange's formula to find the form of $f(x)$. Given

x	0	2	3	6
$f(x)$	648	704	729	792

5

(b) Find the value of $\int_0^\pi \sin x \, dx$ using trapezoidal rule by dividing the range into 10 equal parts. Compare the result with the actual value. 5

Or

8. *(a)* Find a cubic polynomial which takes the set of values (0, 1), (1, 2), (2, 1) and (3, 10). 5

(b) Apply Simpson's $\frac{3}{8}$ rule to evaluate $\int_0^2 \frac{dx}{1+x^3}$ to 2 decimal places by dividing the range into eight equal parts. 5

9. (a) The values of annuities for certain ages are given for the following ages. Find the annuity at age 27·5, using Gauss' forward interpolation formula : 5

Age	25	26	27	28	29
Annuity	16·195	15·919	15·630	15·326	15·006

(b) Find a root of the equation $x^3 - x^2 + x - 7 = 0$ correct to 3 decimal places using the bisection method. 5

Or

10. (a) Find the inverse of the following matrix, using Gauss elimination method : 5

$$\begin{bmatrix} 3 & -1 & 10 & 2 \\ 5 & 1 & 20 & 3 \\ 9 & 7 & 39 & 4 \\ 1 & -2 & 2 & 1 \end{bmatrix}$$

(b) From the following data, find $f(1\cdot6)$, using Newton's forward interpolation formula : 5

x	1	1·4	1·8	2·2
$y = f(x)$	3·49	4·82	5·96	6·5

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