2024

(FYUGP)

(1st Semester)

PHYSICS

(Minor)

Paper Code : PHYM–101(T)

(Mathematical Physics—I)

Full Marks : 75 *Pass Marks* : 40%

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Sketch the graph of $f(x) = e^{-x^2}$. Discuss the behaviour of the function as x approaches $\pm\infty$ and its key features. 5

(b) State the Taylor series expansion of e^x about $x = 0$ up to the x^4 term. 5

2. (a) Solve the first-order differential equation $\frac{dy}{dx} + 3y = 6x$ using the integrating factor method. 5

(b) Solve the homogeneous differential equation $y'' - 4y' + 4y = 0$. 5

UNIT—II

3. (a) Define partial derivatives. For the function $f(x, y) = x^2y + 3xy^2$, calculate the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. 5

(b) State Bayes' theorem. Given that $P(A) = 0{\cdot}5$, $P\left(\frac{B}{A}\right) = 0{\cdot}3$ and $P(B) = 0{\cdot}4$, find $P\left(\frac{A}{B}\right)$. 5

4. (a) Explain exact and inexact differentials. Determine if the differential $M(x, y)\,dx + N(x, y)\,dy$ is exact, where $M(x, y) = x^2 + y$ and $N(x, y) = x + y^2$. 5

(b) Define the Poisson distribution. If the average number of phone calls to a call centre per hour is 8, what is the probability of receiving exactly 10 calls in one hour? 5

UNIT—III

5. *(a)* Define the formula for the scalar triple product of three vectors and discuss its interpretation in terms of volume. 5

   (b) Given the vectors $A = 2\hat{i} + 3\hat{j} - \hat{k}$ and $B = \hat{i} - \hat{j} + 4\hat{k}$, calculate their dot product. Also, show that the dot product is invariant under rotation. 5

6. *(a)* Explain the vector product of two vectors and illustrate its geometric interpretation in terms of area. 5

   (b) Given the vector field $F = 3x\hat{i} + 2y\hat{j} - z\hat{k}$, calculate the divergence and determine whether the field is divergent or not. 5

UNIT—IV

7. *(a)* Explain the concept of infinitesimal line, surface and volume elements. How are they used in vector calculus? 5

   (b) Calculate the line integral of the vector field $F = (2x, 3y, -z)$ along the path C defined by $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. 5

8. (a) Using Stokes' theorem, compute the circulation of the vector field $F = (x^2, y^2, z^2)$ around the boundary of the surface $z = 1 - x^2 - y^2$ within the plane $z = 0$. 5

(b) Show how the flux of a vector field through a closed surface can be related to the divergence of the field using Gauss' divergence theorem. 5

UNIT—V

9. (a) Compare and contrast the expressions for gradient, divergence and curl in Cartesian and spherical coordinates. 5

(b) Find the Laplacian of the function $f = e^{-r}$ in spherical coordinates. 5

10. (a) Discuss the properties of the Dirac delta function and provide examples of its use in integrals. 5

(b) Evaluate the integral

$$\int_{-\infty}^{\infty} \delta(x) e^x dx$$ 5

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Subject Code : Bs/PHYM-101(T)

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA
1st Semester End Term
Examination, **2024 (FYUGP)**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Booklet No. A 91

Date Stamp ....

....

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA
1st Semester End Term
Examination, **2024 (FYUGP)**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

DESCRIPTIVE TYPE

Booklet No. B ....

Signature of Invigilator(s)

/120

2024

(FYUGP)

(1st Semester)

PHYSICS

(Minor)

Paper Code : PHYM–101(T)

(Mathematical Physics—I)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×15=15

1. Which of the following is a necessary condition for a function to be differentiable at $x = a$?

(*a*) The function must be continuous at $x = a$ ( )

(*b*) The function must be bounded at $x = a$ ( )

(*c*) The function must be linear at $x = a$ ( )

(*d*) The function must be differentiable at all points ( )

2. What is the solution to the differential equation $\frac{dy}{dx} = y + 2$?

(a) $y = Ce^{x} - 2$ ( )

(b) $y = Ce^{x} + 2$ ( )

(c) $y = Ce^{-x} + 2$ ( )

(d) $y = Ce^{-x} - 2$ ( )

3. Which of the following series is used to approximate functions of the form $(1+x)^n$ for small x?

(a) Maclaurin series ( )

(b) Fourier series ( )

(c) Binomial series ( )

(d) Taylor series ( )

4. What is the partial derivative of $f(x, y) = x^2y + y^3$ with respect to x?

(a) $2xy$ ( )

(b) $x^2 + 3y^2$ ( )

(c) $2xy + y^3$ ( )

(d) y^3 ( )

5. The variance of a Poisson distribution with $\lambda = 4$ is

(a) 4 ( )

(b) 2 ( )

(c) 1 ( )

(d) 0 ( )

6. For a binomial distribution with $n = 5$ and $p = 0{\cdot}4$, what is the probability of getting exactly 2 successes?

(a) 0·2304 ( )

(b) 0·3840 ( )

(c) 0·4608 ( )

(d) 0·4096 ( )

7. What is the scalar product of two vectors $A = 3\hat{i} + 4\hat{j}$ and $B = 2\hat{i} - \hat{j}$?

(a) 8 ( )

(b) 6 ( )

(c) 14 ( )

(d) 10 ( )

8. The vector product of $A = \hat{i} + 2\hat{j} - \hat{k}$ and $B = 2\hat{i} - \hat{j} + 3\hat{k}$ is

(a) $-2\hat{i} + \hat{j} + \hat{k}$ ( )

(b) $-\hat{i} + 5\hat{j} - 3\hat{k}$ ( )

(c) $\hat{i} - 5\hat{j} + \hat{k}$ ( )

(d) $-\hat{i} - 2\hat{j} + 5\hat{k}$ ( )

9. The Laplacian of the scalar field $\phi = x^2 + y^2 + z^2$ is

(a) 0 ( )

(b) 2 ( )

(c) 4 ( )

(d) 6 ( )

10. Which theorem relates the surface integral of the curl of a vector field over a surface to the line integral of the vector field around the boundary of the surface?

(a) Gauss' divergence theorem ( )

(b) Green's theorem ( )

(c) Stokes' theorem ( )

(d) Fundamental theorem ( )

11. The divergence of a vector field $F = (x, y, z)$ is

(a) $x + y + z$ ( )

(b) $x^2 + y^2 + z^2$ ( )

(c) 0 ( )

(d) 1 ( )

12. The surface integral of $F = (x, y, z)$ over the surface of the unit sphere centred at the origin is

(a) 4π ( )

(b) 2π ( )

(c) π ( )

(d) 0 ( )

13. What is the gradient of the scalar field $\phi = r^2$ in spherical coordinates?

(a) $(2r, 0, 0)$ ( )

(b) $(2r, 2r\sin\theta, 2r\cos\theta)$ ( )

(c) $(r, r\sin\theta, r\cos\theta)$ ( )

(d) $(2r, 2r\cos\theta, 2r\sin\theta)$ ( )

14. The curl of the vector field $F = (x^2, -2xy, z)$ in Cartesian coordinates is

(a) $(2y, -2x, 0)$ ( )

(b) $(-2y, 2x, 0)$ ( )

(c) $(0, 0, 0)$ ( )

(d) $(2y, -2x, 0)$ ( )

15. The integral $\int_{-\infty}^{\infty} \delta(x-a)\,dx$ evaluates to

(a) a ( )

(b) 1 ( )

(c) 0 ( )

(d) $\delta(a)$ ( )

SECTION—II

(*Marks* : 10)

Answer any *five* questions : $2\times5=10$

1. Define the concept of a limit. How is it used to determine the continuity of a function?

2. What is the difference between average and instantaneous rates of change?

3. State the binomial probability formula and explain its applications.

4. What is an integrating factor and how is it used to solve differential equations?

5. Differentiate between scalar fields and vector fields with examples.

6. What is the directional derivative of a scalar field? How does it differ from the gradient?

7. State the importance of the curl of a vector field in understanding the rotation of the field.

8. Calculate the volume integral of the divergence of $F = (x, y, -z)$ over the volume of a unit cube with sides from 0 to 1.

9. What does the curl of a vector field represent in vector calculus?

10. Discuss the significance of the Dirac delta function in physical applications.

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