2025

(FYUGP)

(2nd Semester)

MATHEMATICS

(MINOR)

Paper : MAT/M2

(Ordinary Differential Equations)

Full Marks : 75 *Pass Marks* : 40%

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Find the differential equation of the family of curves, $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants. 4

(b) Find the general solution of $x^5y' + y^5 = 0$. 3

(c) Verify that $yy' = e^{2x}$ is the solution of the differential equation $y^2 = e^{2x} + C$. 3

OR

2. *(a)* Determine whether

$$(y - x^3)\,dx + (x + y^3)\,dy = 0$$

is exact. If it is, solve it. 3

(b) Solve : 3

$$y - x\frac{dy}{dx} = 2\left(y^2 + \frac{dy}{dx}\right)$$

(c) Solve the following by finding an integrating factor : 4

$$(x+2)\sin y\,dx + x\cos y\,dy = 0$$

3. *(a)* Solve : 5

$$x^2 y' - 3xy - 2y^2 = 0$$

(b) Solve : 2½×2=5

(i) $p^2 + p = 6$

(ii) $p^2 - 2xp + 1 = 0$

OR

4. *(a)* Solve : 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

(b) Solve : 5

$$x\left(\frac{dy}{dx}\right) + y^2 x = y$$

5. *(a)* If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that their Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on $[a, b]$. 5

(b) Verify that e^{-x}, e^{3x} and e^{4x} are solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Also show that they are linearly independent on the interval $-\infty < x < \infty$ and write the general solution. 5

OR

6. Find the general solution of the following differential equations : 5+5=10

(i) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0$

(ii) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \sin x$

7. *(a)* Solve $x^2y'' + xy' - y = 0$ given that $x + \dfrac{1}{x}$ is one integral. 5

(b) By reducing to normal form, solve $y'' - 2\tan x\, y' + y = 0$. 5

OR

8. *(a)* Solve $(y''+y)\cot x+2(y'+y\tan x)=\sec x$ by reducing to normal form. 5

(b) Solve the equation $xy''+(1-x)y'-y=e^x$ with the help of operators. 5

9. *(a)* Find the CF and PI of

$$(D^2+31D+240)y=272e^{-x}$$ 5

(b) Given that $y=x$ is a solution of

$$(x^2+1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

Find a linearly independent solution by reducing the order. 5

OR

10. *(a)* Solve : 5

$$(x^2+y^2+e^x)dx+2xy\,dy=0$$

(b) Solve the following by operators : 5

$$xy''+(x-2)y'-2y=x^3$$

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Subject Code : Ba/Bs/MAT/M2

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA 2nd Semester End Term Examination, **2025 (FYUGP)**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s)

Signature of Examiner(s)

Booklet No. A

Date Stamp ....

....

To be filled in by the Candidate

BA / BSc / BCom / BBA / BCA

2nd Semester End Term

Examination, **2025 (FYUGP)**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

DESCRIPTIVE TYPE

Booklet No. B ....

Signature of Invigilator(s)

2025

(FYUGP)

(2nd Semester)

MATHEMATICS

(MINOR)

Paper : MAT/M2

(Ordinary Differential Equations)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

Put a Tick (✓) mark against the correct answer in the brackets provided : $1\times15=15$

1. How many independent variables does an ordinary differential equation have?

(*a*) One ( )

(*b*) Two ( )

(*c*) Three ( )

(*d*) Any number ( )

2. The differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^5 + y = x^2$$

is of

(a) third degree, linear ( )

(b) third degree, non-linear ( )

(c) second order, linear ( )

(d) second order, non-linear ( )

3. The general solution of $y' = 2x$ is

(a) $y = x^2 + C$ ( )

(b) $xy = C$ ( )

(c) $x^2y = C$ ( )

(d) $y = Cx^2$ ( )

4. In an exact differential equation of the form $Mdx + Ndy = 0$

(a) M and N are functions of x only ( )

(b) M and N are functions of y only ( )

(c) M is a function of x and N is a function of y ( )

(d) M and N are functions of x and y ( )

5. The substitution used to solve a homogeneous first-order differential equation is

(a) $x = vy$ ( )

(b) $y = vx$ ( )

(c) $v = x - y$ ( )

(d) $v = x + y$ ( )

6. The differential equation $(x^2y)\,dx + (x^3 + y^3)\,dy = 0$ is

(a) exact ( )

(b) linear ( )

(c) homogeneous ( )

(d) None of the above ( )

7. A Bernoulli equation is a first-order differential equation of the form

(a) $\dfrac{dy}{dx} + P(x)y = Q(x)$ ( )

(b) $\dfrac{dy}{dx} + P(x)y = Q(x)\,y^n$ ( )

(c) $M(x, y)\,dx + N(x, y)\,dy = 0$ ( )

(d) $y'' + P(x)y' + Q(x)y = 0$ ( )

8. The second-order linear differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

is homogeneous, if

(a) $P(x) = 0$ ( )

(b) $Q(x) = 0$ ( )

(c) $R(x) = 0$ ( )

(d) None of the above ( )

9. The particular solution of $y'' + y = 0$, $y(0) = 2$, $y'(0) = 3$ is

(a) $y = 3\cos x - 2\sin x$ ( )

(b) $y = 3\cos x + 2\sin x$ ( )

(c) $y = 3\sin x - 2\cos x$ ( )

(d) $y = 3\sin x + 2\cos x$ ( )

10. The general solution of an *n*th order non-homogeneous linear differential equation is given by

(a) complementary function ( )

(b) particular integral ( )

(c) complementary function + particular integral ( )

(d) Wronskian ( )

11. The differential equation

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = 0$$

where a_0, a_1, $\cdots$, a_n are real constants will have a general solution consisting of

(a) no constant ( )

(b) n constants ( )

(c) $(n-1)$ constants ( )

(d) $(n+1)$ constants ( )

12. The particular integral of $(D^2 + y)y = 4\cos x$ is

(a) $2\log(1+x)\sin\log(1+x)$ ( )

(b) $\log(1+x)\sin\log(1+x)$ ( )

(c) $2\log(1+x)\cos\log(1+x)$ ( )

(d) $\log(1+x)\cos\log(1+x)$ ( )

13. In reduction to normal form

(a) we change the independent variable ( )

(b) we change the dependent variable ( )

(c) we change both the dependent and independent variables ( )

(d) None of the above ( )

14. The complementary function of

$$(D^2 - 3D + 2)y = e^x + e^{2x}$$

is

(a) $C_1 \cos x + C_2 \sin x$ ( )

(b) $C_1 \sin x + C_2 \cos x$ ( )

(c) $C_1 e^x + C_2 e^{2x}$ ( )

(d) $C_1 e^{-x} + C_2 e^{-2x}$ ( )

15. The solution of $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$ is

(a) $x^2 - y^2 = C_1, \quad x^2 - z^2 = C_2$ ( )

(b) $x^2 + y^2 = C_1, \quad x^2 + z^2 = C_2$ ( )

(c) $x - y = C_1, \quad x - z = C_2$ ( )

(d) $x + y = C_1, \quad x + z = C_2$ ( )

SECTION—II

(*Marks* : 10)

Answer any *five* of the following : 2×5=10

1. Solve $p = \log(px - y)$.

2. Find the differential equation of $x^2 + y^2 + 2ax = 0$ by eliminating a.

3. Find the particular integral of $(D^2 - 3D + 2)y = e^{3x}$.

4. The roots of the auxiliary equation corresponding to a certain tenth-order homogeneous linear differential equation with constant coefficients are $4,\ 4,\ 4,\ 4,\ 2+3i,\ 2-3i,\ 2+3i,\ 2-3i,\ 2+3i,\ 2-3i.$ Write the general solution.

5. Find an integrating factor of

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$$

6. If $m_1 \neq m_2$, then

$$y = \frac{e^{m_1 x} - e^{m_2 x}}{m_1 - m_2}$$

is a solution of

$$y'' - (m_1 + m_2) y' + m_1 m_2 \, y = 0$$

Prove or disprove.

7. Solve :

$$\frac{dx}{y^2+z^2}=\frac{dy}{-xy}=\frac{dz}{-xz}$$

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